Intro to Composite Functions

These notes are intended as a summary of section 4.3 (p. 291 – 297) in your workbook. You should also read the section for more complete explanations and additional examples.

Introduction

Cam is a farmer. Each year he plants seeds that turn into corn. The function below gives the amount of corn , *C*, in kilograms, that he expects to produce if he plants corn on *a* acres of land.

$$C(a) = 7500a - 1500$$

For example, if Cam plants two acres, he expects to produce

$$C(2) = 7500(2) - 1500 = 13500\ kg \text{ of corn}$$

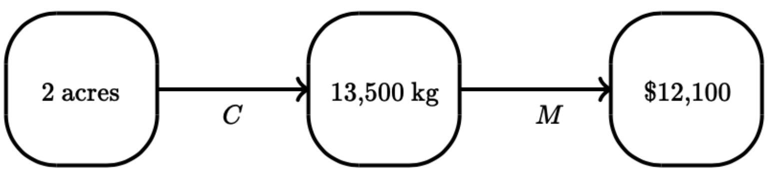
What Cam really wants to know is how much money he will make from selling his corn. He uses the following function to predict the amount of money, *M*, in dollars, that he will earn from selling *c* kilograms of corn.

$$M(c) = 0.9c - 50$$

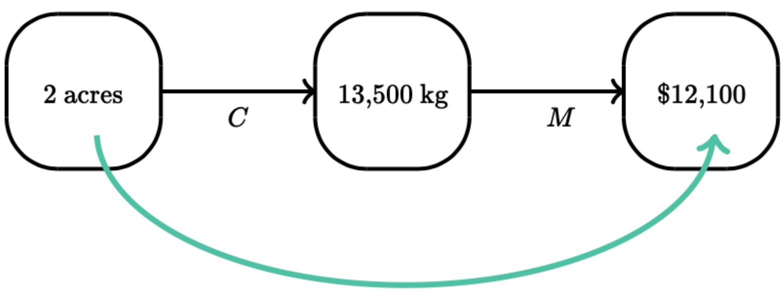
So, if Cam produces 13 500 kg of corn, he can expect to make

$$M(13500) = 0.9(13500) - 50 = \$12100$$

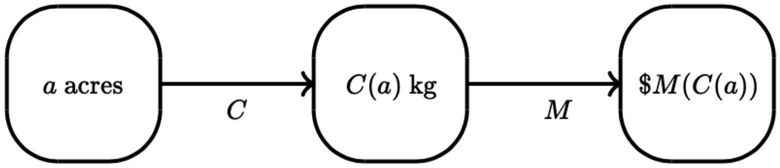
Notice that Cam has to use two separate functions to get from acres planted to expected earnings. The first function, *C*, takes acres to corn, while the second function, *M*, takes corn to money.



Wouldn't it be great if Cam could write a function that predicted the amount of money he would make directly from the number of acres he planted?



If Cam plants corn on a acres, he expects to produce $C(a)$ kilograms of corn. If he produces $C(a)$ kilograms of corn, he expects to make $M(C(a))$ dollars.



So, to find a function that converts a acres directly into expected earnings, we must find the expression $M(C(a))$.

Notice that in the expression $M(C(a))$, the input of the function M is $C(a)$. So, to find this expression, we must substitute $C(a)$ for c in the function M.

$$\begin{aligned} M(c) &= 0.9c - 50 \\ M(C(a)) &= 0.9(C(a)) - 50 \\ &= 0.9(7500a - 1500) - 50 \\ &= 6750a - 1350 - 50 \\ &= 6750a - 1400 \end{aligned}$$

So the function $M(C(a)) = 6750a - 1400$ converts acres planted directly into expected earnings. Let's use this function to predict the amount of money Cam would earn from planting two acres of corn.

$$M(C(2)) = 6750(2) - 1400 = \$12100$$

Cam should expect to earn \$12100, which is consistent with our previous work.

Composition of Functions

What we just found is called a composite function. The composition of two functions, written

$$f(g(x)) \qquad \text{or} \qquad f \circ g(x)$$

is the function that results when $g(x)$ is substituted for every x in $f(x)$.

Note: Both of the expressions above are read as f of g at x.

Evaluating Composite Functions

Composite functions can be evaluated using one of several methods:

To evaluate $f\left(g\left(x\right)\right)$ when $x = a$:

1. Using tables of values

   - use the first table of values to determine the *y*-coordinate of $g\left(x\right)$ when $x = a$
   - use the second table of values to determine the *y*-coordinate of $f\left(x\right)$ when *x* is equal to the *y*-coordinate of $g\left(x\right)$ determined in the first step
   - the *y*-coordinate of $f\left(x\right)$, as determined in the second step, is the value of $f\left(g\left(a\right)\right)$

2. Graphically

   - locate the point on the graph of $g\left(x\right)$ whose *x*-coordinate is *a*
   - locate the point on the graph of $f\left(x\right)$ whose *x*-coordinate is equal to the *y*-coordinate of the point we located on the graph of $g\left(x\right)$
   - the *y*-coordinate of the point we found on $f\left(x\right)$ is the value of $f\left(g\left(a\right)\right)$

3. Algebraically

   - determine the value of $g\left(a\right)$ by substituting *a* for *x* in $g\left(x\right)$
   - determine the value of $f\left(g\left(a\right)\right)$ by substituting the value of $g\left(a\right)$ for *x* in $f\left(x\right)$

Example 1 (sidebar p. 294)
The tables below define two functions.

x	*f (x)*
–2	8
–1	3
0	0
1	–1
2	0

x	*g (x)*
–2	3
–1	2
0	1
1	0
2	–1

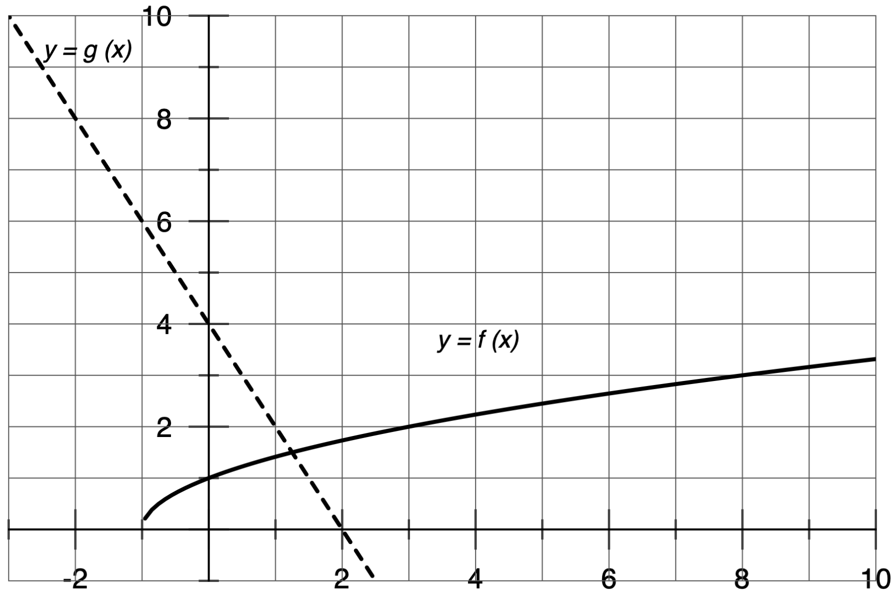
Use these tables to determine each value.

a) $g\left(f\left(2\right)\right)$

b) $g\left(g\left(2\right)\right)$

Example 2 (sidebar p. 295)

Given the graphs of $y = f(x)$ and $y = g(x)$, determine each value below.



a) $f(g(-2))$

b) $g(f(3))$

Example 3 (sidebar p. 296)

Given the functions $f(x) = x^2 + 3x$ and $g(x) = -2x + 1$, determine each value.

a) $f(g(9))$

b) $g(f(9))$

Example 4 (sidebar p. 297)

Given $f(x) = 2x^2 + 1$ and $g(x) = 2x + 7$, determine an explicit equation for each composite function, then state its domain and range.

a) $f(g(x))$

b) $g(f(x))$

c) $g\left(g\left(x\right)\right)$

Homework: #4 – 11, 13, 15 in the section 4.3 exercises (p. 298 – 304). Answers on p. 305.